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Kappa-symmetric derivative corrections to D-brane dynamics

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Abstract: We show how the superembedding formalism can be applied to construct manifestly kappa-symmetric higher derivative corrections for the D9-brane. We also show that all correction terms appear at even powers of the fundamental length scale l. We explicitly construct the first potential correction, which corresponds to the kappa-symmetric version of the $\partial^4 F^4$, which one finds from the four-point amplitude of the open superstring.

Keywords: D-branes, Supersymmetric Effective Theories, Superspaces.

Contents

1. Introduction

The derivation of the effective dynamics of branes in string- and M-theory is a difficult problem. Born-Infeld theory has been shown to describe, in the limit of slowly varying field strengths, the effective dynamics of D-branes to all orders in α' [1, 2]. However, when this limit is not valid, "derivative corrections" to Born-Infeld theory, that is correction terms involving the derivatives of the fields, i.e. the world volume field strength tensor and the transverse location of the brane, must be taken into account. The first such correction term was calculated in [3]. In recent years, there has been some progress in the construction of corrections, see for example [4, 5]. Also, more recently, there have been several approaches based on supersymmetry. In [6, 7], for example, the leading supersymmetric correction for the gauge field in the abelian case, which is the supersymmetric completion of $\partial^4 F^4$, has been derived. The full supersymmetric four-point function was constructed in [8]. The leading correction to the non-abelian theory, as a description of coinciding D-branes, has also been identified using various methods, see [9, 10, 11, 12, 13, 14, 15, 7]. A non-abelian generalisation of the four-point function of [8] was discussed in [16]. However, all these results do not possess the κ-symmetry of the undeformed D-brane actions [17, 18].

The superembedding formalism is a manifestly κ-symmetric method for deriving brane dynamics. It was developed in [20, 21, 22][1] and extended into a general framework for the description of superbranes in [24]–[28]. Superembeddings have been used to construct higher derivative brane actions [29, 30]. For the M2-brane, which has no gauge degrees

[1] Note that the concept of source and target superspace appeared already in [19]. For a review of superembeddings see [23].

of freedom, the first potential κ-symmetric correction term has been constructed in [31], using a (deformed) superembedding. In the present work, we use an approach similar to that developed in [31]. We draw on the concept of spinorial cohomologies [10, 11, 32, 33] to construct the κ-symmetric equations of motion corresponding to the $\partial^4 F^4$-term for the D9-brane. We prove that it is the first such potential derivative correction.

The dynamics of the D9-brane are captured by two systems of equations which are linked: the torsion equation, which gives the world-volume torsion of the brane in terms of the pullback of the target-space torsion, and the world-volume Bianchi identities for the gauge field living on the brane. Dimensional analysis puts a natural constraint on the components of the field strength super 2-form, the so-called $\mathcal{F}$-constraint. This constraint implies Born-Infeld dynamics for the D9-brane. The introduction of a fundamental length scale allows us to deform the $\mathcal{F}$-constraint and thereby the Bianchi identities. This leads to derivative corrections in the equations of motion. This approach is the gauge-field analogue of that used in [31], in which the first potential derivative correction for the M2-brane was constructed by deforming the constraint on transverse degrees of freedom, the "embedding constraint".

Section 2 summarises the superembedding approach as applied to the D9-brane. This section essentially contains material covered before [34]–[37], but serves to prepare the ground for the discussion of derivative corrections in this approach.

In section 3 we prove generally that deformations of D9-dynamics and also $D = 10$ SYM (or, in fact $D = 10$ Yang-Mills coupled to a spinor of canonical dimension), can only exist with coefficients of even powers of the fundamental length scale l, or, if one likes, integer powers of α'. We then explicitly deform the standard constraints on the D9-brane in powers of the fundamental length scale l. We show that the first potential deformation allowed, which is at l^2, does not exist. We then explicitly construct the deformation at the next order, l^4, cubic in fields. As in the case of the M2-brane, the problem turns out to be a spinorial cohomology problem as described in [10, 11]. We find a unique solution cubic in fields (but have not checked consistency at higher orders in fields or higher powers of l).

In section 4 we show the effect of the deformed constraint on the kappa-symmetric equations of motion for the worldvolume theory.

We conclude with some comments on higher order terms and other branes.

2. The D9-brane as a superembedding

Superembeddings are the generalisation of surface theory to supermanifolds. They are well suited for describing gauge invariant brane dynamics for the branes which arise in superstring theories and M-theory. The fermionic gauge symmetry, kappa-symmetry, which is present in the Green-Schwarz description of such objects has a natural geometrical interpretation as the odd part of the local reparametrisation invariance of the embedded manifold (the brane). The parameter of the kappa symmetry in the Green-Schwarz formalism is replaced by an odd vector field on the worldvolume of the brane.

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We consider such an embedding, with $\mathcal{M}$ labelling the worldvolume of a p-brane and $\underline{\mathcal{M}}$ labelling the D dimensional target space, which we take to be flat. The embedding is a map,

$$f : \mathcal{M} \longrightarrow \underline{\mathcal{M}} \,. \tag{2.1}$$

The cotangent frame on the target space, $E^{\underline{A}} = (E^{\underline{a}}, E^{\underline{\alpha}})$ can be pulled back to the worldvolume via the pullback map, f^*, and expressed in terms of the cotangent frame on the worldvolume, $E^A = (E^a, E^\alpha)$,

$$f^* E^{\underline{A}} = E^A E_A{}^{\underline{A}} \,. \tag{2.2}$$

We use the convention that Latin indices refer to the bosonic directions and Greek indices to the fermionic directions. Capital letters are used for both, $A = (a, \alpha)$. Underlined indices refer to target space quantities while those without an underline refer to the worldvolume. Target space indices are split by the embedding into directions tangential and normal to the worldvolume. Normal directions are denoted with a primed index, a' or α'. The matrix $E_A{}^{\underline{A}}$ is called the embedding matrix and contains the geometrical information of the embedding. It can be put into a general form,

$$E_A{}^{\underline{A}} = \begin{pmatrix} u_a{}^{\underline{a}} & \Lambda_a{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} \\ \Psi_\alpha{}^{b'} u_{b'}{}^{\underline{a}} & u_\alpha{}^{\underline{\alpha}} + h_\alpha{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} \end{pmatrix} . \tag{2.3}$$

Here the matrix $u_{\underline{a}}{}^{\underline{b}}$ is an element of $\mathrm{SO}(1, D-1)$ and $u_{\underline{\alpha}}{}^{\underline{\beta}}$ is the corresponding element of $\mathrm{Spin}(1, D-1)$. For certain superembeddings, the standard constraint

$$E_\alpha{}^{\underline{a}} = 0 \,, \tag{2.4}$$

implies the equations of motion for the worldvolume supermultiplet. Embeddings describing the M2-brane and M5-brane are examples of this. For some superembeddings of low bosonic codimension, a second constraint, called the $\mathcal{F}$-constraint, is required. In the D-brane superembeddings there is a closed super three-form, $\underline{H}$, in the type-II supergravity background. One introduces a worldvolume two-form, $\mathcal{F}$, which satisfies $d\mathcal{F} = f^* \underline{H}$. The standard $\mathcal{F}$-constraint takes the form

$$\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\alpha b} = 0 \,. \tag{2.5}$$

In the absence of an explicit length scale, with the standard embedding condition, no objects of negative mass dimension appear. Dimensional analysis then forces the above form of the $\mathcal{F}$-constraint. The constraint implies that the multiplet on the worldvolume of the D-brane is given by the Maxwell supermultiplet (which is on-shell) and thus the fields are constrained to satisfy their equations of motion. One can write the two-form, $\mathcal{F}$, as

$$\mathcal{F} = F + f^* \underline{B} \,. \tag{2.6}$$

Here, $\underline{B}$, is the two-form potential for $\underline{H}$ in the background, $d\underline{B} = \underline{H}$, and F is a two-form field strength for the worldvolume one-form gauge potential, A, satisfying $dA = F$.

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In a flat background, one can give an explicit solution for $\underline{B}$ in terms of the target space coordinates. The deformation of the standard Maxwell constraints on the components of F, which defines supersymmetric (Born-Infeld) theory living on the worldvolume of the D-brane, can then readily be deduced from the $\mathcal{F}$-constraint. This was carried out explicitly for the D9-brane of IIB in [37].

In [31] a deformation of the embedding constraint was used to describe higher derivative corrections to the M2-brane equations of motion. Here we will be constructing derivative deformations of D9-brane dynamics. In the case of the D9-brane (and other space-filling branes), the embedding condition is satisfied without loss of generality (so Ψ in (2.3) is 0) since there are no normal bosonic directions. Thus the embedding matrix must remain undeformed even when derivative corrections are to be included. Instead, one must deform the $\mathcal{F}$-constraint. Note that for space filling branes, we can also always take the matrix u in (2.3) to be the identity without loss of generality, thus identifying the bosonic cotangent frame of the two supermanifolds. We therefore choose for our D9-brane embedding

$$E_A{}^{\underline{A}} = \begin{pmatrix} \delta_a{}^{\underline{a}} & \Lambda_a{}^{\beta'}\delta_{\beta'}{}^{\underline{\alpha}} \\ 0 & \delta_\alpha{}^{\underline{\alpha}} + h_\alpha{}^{\beta'}\delta_{\beta'}{}^{\underline{\alpha}} \end{pmatrix}. \tag{2.7}$$

Here, and from now on, the Greek indices α are 16 component Majorana-Weyl spinor indices of Spin$(1,9)$. The two chiralities are denoted by upstairs and downstairs indices. The target space indices $\underline{\alpha}$ can be split into the pair αi where the index i is an SO(2) index.

The main aim of this paper is to describe how one can systematically deform the $\mathcal{F}$-constraint to include derivative corrections to Born-Infeld theory. The deformations are described in terms of spinorial cohomology and will be discussed in the next section.

We now focus on the supergeometry of the D9-brane superembedding and show how one can deduce the equations of motion given the standard $\mathcal{F}$-constraint . We consider the embedding of $N=(1,0)$, $D=10$ superspace into flat $N=(2,0)$, $D=10$ superspace. The worldvolume geometry is constrained by the torsion equation which gives the worldvolume torsion in terms of the target space torsion. We have

$$T^A E_A{}^{\underline{A}} = dE^A E_A{}^{\underline{A}} = f^* dE^{\underline{A}} = f^* T^{\underline{A}}. \tag{2.8}$$

The target space will be assumed flat throughout and hence the only non-zero component of the target space torsion is at dimension zero,

$$T_{\alpha i \beta j}{}^{c} = -i\delta_{ij}(\gamma^c)_{\alpha\beta}\,. \tag{2.9}$$

In components the torsion equation reads,

$$\nabla_A E_B{}^{\underline{C}} - (-1)^{AB}\nabla_B E_A{}^{\underline{C}} + T_{AB}{}^{C} E_C{}^{\underline{C}} = (-1)^{A(B+\underline{B})} E_B{}^{\underline{B}} E_A{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}}. \tag{2.10}$$

Analysing this equation level by level in dimension and substituting (2.7), we find:
Dimension 0:

$$T_{\alpha\beta}{}^{c} = -i(\delta_\alpha^{\gamma}\delta_\beta^{\gamma} + h_\alpha{}^{\gamma} h_\beta{}^{\delta})(\gamma^c)_{\gamma\delta}\,. \tag{2.11}$$

Dimension $1/2$:

$$T_{\alpha\beta}{}^{\gamma} = 0\,, \tag{2.12}$$

$$T_{\alpha b}{}^{c} = i\Lambda_b{}^{\beta} h_\alpha{}^{\gamma} (\gamma^c)_{\gamma\beta}\,, \tag{2.13}$$

$$2\nabla_{(\alpha} h_{\beta)}{}^{\gamma} = -T_{\alpha\beta}{}^{a} \Lambda_a{}^{\gamma}\,. \tag{2.14}$$

Dimension 1:

$$T_{\alpha b}{}^{\gamma} = 0\,, \tag{2.15}$$

$$T_{ab}{}^{c} = -i\Lambda_b{}^{\beta} \Lambda_a{}^{\alpha} (\gamma^c)_{\alpha\beta}\,, \tag{2.16}$$

$$\nabla_\alpha \Lambda_b{}^{\gamma} - \nabla_b h_\alpha{}^{\gamma} = -i\Lambda_b{}^{\beta} \Lambda_c{}^{\gamma} h_\alpha{}^{\delta} (\gamma^c)_{\beta\delta}\,. \tag{2.17}$$

dimension $3/2$:

$$T_{ab}{}^{\gamma} = 0\,, \tag{2.18}$$

$$2\nabla_{[a} \Lambda_{b]}{}^{\gamma} = i\Lambda_b{}^{\beta} \Lambda_a{}^{\alpha} \Lambda_c{}^{\gamma} (\gamma^c)_{\alpha\beta}\,. \tag{2.19}$$

Next we proceed to the Bianchi identity for the two-form, $\mathcal{F}$,

$$d\mathcal{F} = f^* \underline{H}\,. \tag{2.20}$$

The non-zero components of the three-form, $\underline{H}$, in a flat IIB background, are given by

$$H_{\alpha i \beta j c} = -i(\gamma_c)_{\alpha\beta} H_{ij}\,. \tag{2.21}$$

There are two linearly independent closed forms, given by

$$H_{ij} = (\tau^1)_{ij} \text{ and } H_{ij} = (\tau^3)_{ij}\,, \tag{2.22}$$

with τ^i being the Pauli matrices. We choose the first of these solutions, $H_{ij} = (\tau^1)_{ij}$.

In components, the Bianchi identity reads

$$3\nabla_{[A} \mathcal{F}_{BC]} + 3T_{[AB}{}^{D} \mathcal{F}_{|D|C]} = (-1)^{A(B+\underline{B}+C+\underline{C})+B(C+\underline{C})} E_C{}^{\underline{C}} E_B{}^{\underline{B}} E_A{}^{\underline{A}} H_{\underline{ABC}}\,. \tag{2.23}$$

Again, we analyse this level by level in dimension.

Dimension $-1/2$:

$$\nabla_{(\alpha} \mathcal{F}_{\beta\gamma)} + T_{(\alpha\beta}{}^{d} \mathcal{F}_{d\gamma)} = 0\,. \tag{2.24}$$

Dimension 0:

$$2\nabla_{(\alpha} \mathcal{F}_{\beta)c} + \nabla_c \mathcal{F}_{\alpha\beta} + T_{\alpha\beta}{}^{d} \mathcal{F}_{dc} + 2T_{(\alpha c}{}^{d} \mathcal{F}_{d\beta)} = E_\alpha{}^{\underline{\alpha}} E_\beta{}^{\underline{\beta}} H_{\underline{\alpha}\underline{\beta}c}\,. \tag{2.25}$$

Dimension $1/2$:

$$\nabla_\alpha \mathcal{F}_{bc} - 2\nabla_{[b} \mathcal{F}_{\alpha c]} + 2T_{\alpha[b}{}^{d} \mathcal{F}_{|d|c]} + T_{bc}{}^{d} \mathcal{F}_{d\alpha} = -2E_{[b}{}^{\underline{\beta}} E_\alpha{}^{\underline{\alpha}} H_{\underline{\alpha}\underline{\beta}c]}\,. \tag{2.26}$$

Dimension 1:

$$\nabla_{[a} \mathcal{F}_{bc]} + T_{[ab}{}^{d} \mathcal{F}_{|d|c]} = E_{[b}{}^{\underline{\beta}} E_a{}^{\underline{\alpha}} H_{\underline{\alpha}\underline{\beta}c]}\,. \tag{2.27}$$

If we impose the standard constraints on $\mathcal{F}$ given by

$$\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\alpha b} = 0 \tag{2.28}$$

then the Bianchi identity at dimension $-1/2$ is automatically satisfied. We will deform the $\mathcal{F}$-constraint in the next section but for now we work with the undeformed constraint.

If we linearise the above system of equations we find that the solution can be written quite simply. We can split Λ into its two irreducible representations,

$$\Lambda_a{}^\alpha = (\gamma_a)^{\alpha\beta}\psi_\beta + \hat{\Lambda}_a{}^\alpha\,, \tag{2.29}$$

where $\hat{\Lambda}$ is gamma-traceless. Then, we substitute our results from the torsion equation into the Bianchi identity. From the dimension 0 component we find, by contracting with $(\gamma^1)^{\alpha\beta}$ and $(\gamma^5)^{\alpha\beta}$

$$h_\alpha{}^\beta = \frac{1}{4}\mathcal{F}_{cd}(\gamma^{cd})_\alpha{}^\beta. \tag{2.30}$$

At dimension $1/2$ we find

$$\psi_\delta = 0\,, \tag{2.31}$$

$$\nabla_\alpha \mathcal{F}_{bc} = 2i\hat{\Lambda}_{[b}{}^\beta(\gamma_{c]})_{\alpha\beta}\,. \tag{2.32}$$

From the dimension 1 Bianchi identity we get

$$\nabla_\alpha \hat{\Lambda}_b{}^\beta = \frac{1}{4}\nabla_b \mathcal{F}_{cd}(\gamma^{cd})_\alpha{}^\beta \tag{2.33}$$

$$\nabla_{[a}\mathcal{F}_{bc]} = 0 \tag{2.34}$$

$$\nabla^a \mathcal{F}_{ab} = 0\,. \tag{2.35}$$

The dimension $3/2$ Torsion equation contains

$$\nabla_{[a}\hat{\Lambda}_{b]}{}^\alpha = 0 \tag{2.36}$$

$$\nabla^a \hat{\Lambda}_a{}^\alpha = 0\,. \tag{2.37}$$

The relation (2.31) is the linearised fermionic equation of motion for the worldvolume theory. One can see this by going to static gauge, which is defined by the identification of the worldvolume coordinates x^a and θ^α with the target space coordinates $x^{\underline{a}}$ and $\theta^{\alpha 1}$. The second spinorial coordinate becomes the field on the worldvolume, $\theta^{\alpha 2} = \zeta^\alpha(x^a, \theta^\alpha)$, which is (at lowest order) the spinor field strength superfield of the deformed Maxwell theory. In this gauge, $\Lambda_a{}^\alpha = \nabla_a \zeta^\alpha$, so, at the linear level, the above equation is the Dirac equation for ζ. The full non-linear equation is the supersymmetric Born-Infeld equivalent. The relation (2.35) is the linearised bosonic equation of motion for the for the worldvolume theory. This obviously has the form of the Maxwell equation for the vector field A_a.

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The linear relations constrain the fields $\mathcal{F}_{ab}$ and $\hat{\Lambda}_a{}^\alpha$ and their derivatives to lie in certain representations of Spin$(1,9)$ (see the appendix for notation)

$$\nabla_{a_1}...\nabla_{a_n}\mathcal{F}_{bc} \text{ in the irrep } (n1000)\,, \tag{2.38}$$

$$\nabla_{a_1}...\nabla_{a_{n-1}}\Lambda_{a_n}{}^\alpha \text{ in the irrep } (n0010)\,. \tag{2.39}$$

The non-linear corrections to the linearised equations can then be regarded as an expansion in the above objects (i.e. the fields satisfying their lowest order relations). This is also how we will treat the additional non-linear corrections which appear when we deform the $\mathcal{F}$-constraint.

The non-linear corrections to the undeformed theory can be derived from the system of torsion equations and Bianchi identities. We can decompose the matrix, $h_\alpha{}^\beta$, into irreducible representations,

$$h_\alpha{}^\beta = h\delta_\alpha^\beta + h_{ab}(\gamma^{ab})_\alpha{}^\beta + h_{abcd}(\gamma^{abcd})_\alpha{}^\beta\,. \tag{2.40}$$

As we have seen, at the linearised level, only the two-form component of $h_\alpha{}^\beta$ is non-zero and that this is the two-form component of $\mathcal{F}$, up to a factor. The non-linear parts of $h_\alpha{}^\beta$ will be denoted by $h'_\alpha{}^\beta$ and up to cubic order in $\mathcal{F}$ we find

$$h' = 0\,, \tag{2.41}$$

$$h'_{ab} = \frac{1}{8}(\mathcal{F}_{ac}\mathcal{F}^{cd}\mathcal{F}_{db} + \frac{1}{4}\mathcal{F}_{cd}\mathcal{F}^{cd}\mathcal{F}_{ab})\,, \tag{2.42}$$

$$h'_{abcd} = \frac{1}{16.2.6.4!}\epsilon_{abcdefghij}\mathcal{F}^{ef}\mathcal{F}^{gh}\mathcal{F}^{ij}\,. \tag{2.43}$$

The equations at dimension 1/2 imply,

$$\nabla_\alpha\mathcal{F}_{bc} = 2i\Lambda_{[b}{}^\beta(\gamma_{c]})_{\alpha\beta} - 2i(\gamma^d)_{\gamma\beta}h_\alpha{}^\gamma\mathcal{F}_{d[c}\Lambda_{b]}{}^\beta\,. \tag{2.44}$$

This relation is the supervariation of $\mathcal{F}$ in the non-linear theory. Note that the first term contains a linear piece and also a non-linear piece (the gamma-trace), given by

$$\psi_\delta = \frac{1}{10}(\gamma^a)_{\alpha\delta}\Lambda_a{}^\alpha = \frac{1}{700}\Big[-6(\gamma^d)_{\beta\gamma}h_\alpha{}^\gamma\mathcal{F}_{dc}\Lambda_b{}^\beta(\gamma^{bc})^\alpha{}_\delta - 2i(\gamma_a)^{\alpha\beta}\nabla_\alpha h'_\beta{}^\gamma(\gamma^a)_{\gamma\delta} - (\gamma_a)^{\alpha\beta}h_\alpha{}^\epsilon h_\beta{}^\eta(\gamma^c)_{\epsilon\eta}\Lambda_c{}^\gamma(\gamma^a)_{\gamma\delta}\Big] \tag{2.45}$$

This is the non-linear fermionic equation of motion for the worldvolume theory. One can use this equation to determine ψ order by order in fields once $h_\alpha{}^\beta$ is known.

At dimension one we have

$$\nabla_{[a}\mathcal{F}_{bc]} = -i\Lambda_{[b}{}^\beta\Lambda_a{}^\alpha(\gamma^d)_{\alpha\beta}\mathcal{F}_{c]d} \tag{2.46}$$

$$\nabla^b\mathcal{F}_{ab} = \frac{1}{8}[10(\gamma_a)^{\alpha\epsilon}\nabla_\alpha\psi_\epsilon - \nabla_b h'_\alpha{}^\gamma(\gamma^b\gamma_a)_\gamma{}^\alpha + i\Lambda_b{}^\beta\Lambda_c{}^\gamma h_\alpha{}^\delta(\gamma^c)_{\beta\delta}(\gamma^b\gamma_a)_\gamma{}^\alpha]\,, \tag{2.47}$$

which are, respectively, the non-linear Bianchi identity for $\mathcal{F}_{ab}$ and the non-linear bosonic equation of motion. We also find the non-linear supervariation of $\Lambda_a{}^\beta$,

$$\nabla_\alpha\Lambda_a{}^\beta = \nabla_b h_\alpha{}^\gamma - i\Lambda_b{}^\beta\Lambda_c{}^\gamma h_\alpha{}^\delta(\gamma^c)_{\beta\delta}\,. \tag{2.48}$$

These equations determine everything as a series in the the fields of the linearised theory, i.e. $\mathcal{F}_{ab}$ and $\hat{\Lambda}_a{}^\alpha$ and their derivatives, in the representations (2.38), (2.39). Thus we have seen that the $\mathcal{F}$-constraint gives the full non-linear dynamics of the D9-brane worldvolume theory.

Kappa symmetry in this formulation is manifest. The equations we have written down are invariant under diffeomorphisms of the worldvolume. The odd diffeomorphisms are precisely the kappa symmetry transformations [21, 24, 25, 28].

3. Derivative corrections

Our aim is to understand how to introduce a deformation into the theory which will allow a manifestly kappa-symmetric treatment of higher derivative terms. We will follow the analysis presented in [31], where derivative corrections to the worldvolume theory of the eleven-dimensional supermembrane are discussed in the superembeddings framework. In the supermembrane setting, such corrections can be introduced via a deformation of the embedding condition $E_\alpha{}^{\underline{a}} = 0$. In contrast, the only freedom we have here is to relax the standard constraint on the worldvolume 2-form $\mathcal{F}$. Thus we have to allow the components $\mathcal{F}_{\alpha\beta}$ (dimension -1) and $\mathcal{F}_{\alpha b}$ (dimension $-1/2$) to be given in terms of the covariant degrees of freedom $\mathcal{F}_{ab}$ (dimension 0) and $\Lambda_a{}^\alpha$ (dimension $1/2$), and their derivatives. We must introduce a parameter, l, with unit negative dimension in order to respect the negative dimensionality of the components of $\mathcal{F}$. We then search for possible deformations of $\mathcal{F}_{\alpha\beta}$ and $\mathcal{F}_{\alpha b}$ order by order in l.

In addition to the constraints described in the previous section, the deformations are constrained by the Torsion equation and Bianchi identity which must still be satisfied. Furthermore, there is a degree of redundancy in these quantities which can be absorbed by field redefinitions, of which there are two types. Firstly, we can redefine the embedding coordinates

$$z^{\underline{M}} \longrightarrow z^{\underline{M}} + (\delta z)^{\underline{M}} . \tag{3.1}$$

This transformation is, equivalently, a target space diffeomorphism. The second type of redefinition is a shift of the one-form potential

$$A \longrightarrow A + \delta A . \tag{3.2}$$

These redefinitions can be used to remove the gamma-trace part of $\mathcal{F}_{\alpha b}$. The quickest way to see this is as follows.[2] When we perform a diffeomorphism of the target space by a vector field v, we see that the three-form $\underline{H}$, changes by

$$\mathcal{L}_v \underline{H} = (i_v d + d i_v)\underline{H} = d i_v \underline{H} . \tag{3.3}$$

[2] We thank Paul Howe for this neat argument.

The second equality follows from the closure of $\underline{H}$. This can be thought of as a change in the two-form potential $\underline{B}$, by $i_v\underline{H}$. Examining the pullback of $\underline{B}$ we see that there is a change (at lowest order) to the quantity $\mathcal{F}_{\alpha b}$

$$\mathcal{F}_{\alpha b} \longrightarrow \mathcal{F}_{\alpha b} + (\gamma_b)_{\alpha\beta} v^{\beta 2} , \tag{3.4}$$

if we choose $v^{\underline{a}} = v^{\alpha 1} = 0$. Therefore such field redefinitions can be used to remove the gamma-trace part of $\mathcal{F}_{\alpha b}$.

The second type of field redefinition is of the same form as one has in the problem of deforming $N = 1$, $D = 10$ Yang-Mills theory [10]. There are two parts to the shift in the one-form potential, the vector part $(\delta A)_a$, and the spinor part $(\delta A)_\alpha$. In the Yang-Mills case the vector shift can be used to remove the vector part of $F_{\alpha\beta}$, leaving only the anti-self-dual 5-form part. Similarly, we can use it here to remove the vector part of $\mathcal{F}_{\alpha\beta}$ and so the deformations we are looking for are also given by anti-self-dual 5-forms J:

$$\mathcal{F}_{\alpha\beta} = J^{abcde} \left(\gamma_{abcde}\right)_{\alpha\beta} . \tag{3.5}$$

In [10], it was noted that the Bianchi identity together with the existence of the spinorial field redefinitions $(\delta A)_\alpha$, implied that each deformation can be identified with an element of a particular spinorial cohomology. This can be seen as follows.

The relevant sequence is one of irreducible representations of Spin$(1, 9)$

$$(00000) \xrightarrow{\Delta_0} (00001) \xrightarrow{\Delta_1} (00002) \xrightarrow{\Delta_2} (00003) \xrightarrow{\Delta_3} \cdots \longrightarrow (0000n) \xrightarrow{\Delta_n} \cdots . \tag{3.6}$$

The irreps are respectively a scalar, a downstairs spinor, an anti-self-dual 5-form, an anti-self-dual 5-form spinor, etc. The operations, Δ_n, are given by the action of a spinorial derivative followed by projection onto the irrep $(0000\, n+1)$. It follows from the algebra of spinorial derivatives that there is a nilpotence condition, given by

$$\Delta_{n+1}\Delta_n = 0 . \tag{3.7}$$

In the case of the D9-brane, the algebra of spinorial derivatives is given by the world-volume torsion. At lowest order in fields this is the standard torsion and so one can use the same arguments as in the $N = 1$, $D = 10$ Yang-Mills case [10] to classify the deformations.

The nilpotence condition allows the definition of the cohomology of Δ_n by

$$\mathcal{H}^n = \frac{\mathrm{Ker}\Delta_n}{\mathrm{Im}\Delta_{n-1}} . \tag{3.8}$$

We find from the dimension $-\frac{1}{2}$ Bianchi identity 2.24 that the anti-self-dual five-form, $\mathcal{F}_{\alpha\beta}$, must satisfy the condition,

$$\nabla_\gamma J_{abcde} - (\gamma_{f[a})_\gamma{}^\alpha \nabla_\alpha J_{bcde]}{}^f - \frac{1}{2}(\gamma_{fg[ab})_\gamma{}^\alpha \nabla_\alpha J_{cde]}{}^{fg} = 0 . \tag{3.9}$$

This condition is the statement that the (00003) representation contained in the quantity $\nabla_\alpha J_{abcde}$ vanishes, i.e.

$$J \in \mathrm{Ker}\Delta_2 . \tag{3.10}$$

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We call this the closure condition. There are some J for which the closure condition is satisfied trivially. Such J correspond to redefinitions, δA, of the spinorial part of the gauge potential 1-form and as such do not correspond to genuine deformations. They are trivially closed because they are given by $J = \Delta_1 \delta A$ and closure follows from the nilpotence condition. Such J are referred to as exact. It therefore follows that the genuine deformations are given elements of the cohomology $\mathcal{H}^2$. We will explicitly calculate such objects.

We will be considering an expansion in numbers of fields. Our aim is calculate the first allowed non-zero J which is a genuine perturbation of the Born-Infeld theory. The objects available to construct J are the same degrees of freedom present in the linearised theory and they can be taken to satisfy their linearised equations of motion and supersymmetry transformation rules since these are true up to higher orders in fields. We therefore have $\Lambda_a{}^\alpha$ and the field strength tensor $\mathcal{F}_{ab}$ and their derivatives, in the representations constrained by the linearised theory (2.38), (2.39).

3.1 All order constraints from dimensional analysis

We have identified deformations of the Born-Infeld theory with anti-self-dual 5-forms, J, of mass dimension -1. The mass dimensions of the quantities from which we can build such J are:

$$[l] = -1 \tag{3.11}$$
$$[\partial_a] = 1 \tag{3.12}$$
$$[\hat{\Lambda}_a{}^\alpha] = \frac{1}{2} \tag{3.13}$$
$$[\mathcal{F}_{ab}] = 0\,. \tag{3.14}$$

The only quantity that comes with a non-integer mass dimension is the fermion Λ (mass dim. 1/2). A deformation of $\mathcal{F}_{\alpha\beta}$ (mass dim. -1) of fractional mass dimension would therefore have to contain an odd number of Λs. However, $\mathcal{F}_{\alpha\beta}$ is a boson, and hence must contain an even number of fermions. Therefore, all deformations come at integer powers in mass dimension.

Any deformation can be written, schematically, as

$$\mathcal{F}_{\alpha\beta} = J^{abcde}\,(\gamma_{abcde})_{\alpha\beta} = l^x\,\partial^k\,\Lambda^{2n}\,\mathcal{F}^m\,. \tag{3.15}$$

Apart from derivatives and the fields Λ and $\mathcal{F}$, the expression can also contain the volume form ϵ^{10} and the metric η which carry an even number of vector indices. Dimensional analysis of the above equation implies

$$-1 = -x + k + n\,. \tag{3.16}$$

For x odd, this implies that $k+n$ is even. Each pair of spinor indices can be replaced with an odd number (which we call p_i for the ith pair) of vector indices by contraction with γ^a, γ^{abc} or γ^{abcde}. The number of vector indices is then

$$k + 2n + \sum_{i=1}^{n} p_i + 2m = (k+n) + \sum_{i=1}^{n} (p_i + 1) + 2m = \text{even}\,, \tag{3.17}$$

and hence there is no way to construct a 5-form, which has an odd number of vector indices.

Essentially the same argument also goes through for any deformations of the (abelian or non-abelian) $N = 1$ Super Yang-Mills lagrangian in $D = 10$ (or indeed Yang-Mills coupled to a spinor of canonical dimension). In this case the deformation is a scalar constructed from the fields W^α (dimension $-1/2$) and F_{ab} (dimension 0) and their covariant derivatives. Again one finds only even integer powers of l (or integer powers of α') in the deformations. Note that this is not the case in the case of the deformed M2-brane [31] where one has to check explicitly that there are no deformations at l^3, for example.

3.2 Perturbative construction of deformations

To construct J we need to find a bosonic anti-self-dual 5-form (rep (00002)) inside the tensor product of some number of the representations (2.38), (2.39). There are none linear in the above objects (obviously) and nor are there any quadratic. Therefore we consider terms cubic in the fields. The first such objects are at order l^2. There are three of the form $\Lambda\Lambda\mathcal{F}$ and one of the form $\nabla\mathcal{F}\mathcal{F}\mathcal{F}$:

$$J_1 = l^2 \Lambda^\alpha_{[a_1} \Lambda_{a_2}{}^\beta \mathcal{F}_{a_3 a_4} (\gamma_{a_5]})_{\alpha\beta} \,, \tag{3.18}$$
$$J_2 = l^2 \Lambda_b{}^\alpha \Lambda^{b\beta} \mathcal{F}_{[a_1 a_2} (\gamma_{a_3 a_4 a_5]})_{\alpha\beta} \,, \tag{3.19}$$
$$J_3 = l^2 \Lambda_b{}^\alpha \Lambda_{[a_1}{}^\beta \mathcal{F}^b{}_{a_2} (\gamma_{a_3 a_4 a_5]})_{\alpha\beta} \,, \tag{3.20}$$
$$J_4 = l^2 \nabla_b \mathcal{F}_{[a_1 a_2} \mathcal{F}^b{}_{a_3} \mathcal{F}_{a_4 a_5]} \,, \tag{3.21}$$

where anti-self-duality is implicit. To see which of these are exact, we must consider all possible spinors $(\delta A)_\alpha$ which are cubic in the fields at order l^2. There are two:

$$\delta A_1 = l^2 \Lambda_a{}^\beta \mathcal{F}^{ab} \mathcal{F}_{bc} (\gamma^c)_{\alpha\beta} \,, \tag{3.22}$$
$$\delta A_2 = l^2 \Lambda_a{}^\beta \mathcal{F}^{ab} \mathcal{F}^{cd} (\gamma_{bcd})_{\alpha\beta} \,. \tag{3.23}$$

To calculate $\Delta_1 \delta A$, we form the quantity $(\gamma_{a_1 \cdots a_5})^{\alpha\beta} \nabla_\alpha \delta A_\beta$ for both of the above. The results are combinations of the Js above. We find that

$$\Delta_1 \delta A_1 = J_2 \,, \tag{3.24}$$
$$\Delta_1 \delta A_2 = J_1 - \frac{1}{4} J_4 \,. \tag{3.25}$$

So the above combinations of Js are exact. To check whether the remaining two linearly independent combinations are closed (i.e. $\Delta_2 J = 0$) we need to consider the possible anti-self-dual 5-form spinors, C, (rep (00003)) which are cubic in the fields at order l^2. There are two of these, given by the (00003) parts of:

$$C_1 = l^2 \Lambda_b{}^\alpha \Lambda_{[a_1}{}^\beta \Lambda_{a_2}{}^\gamma (\gamma_{a_3 a_4 a_5]})_{\alpha\beta} (\gamma^b)_{\gamma\delta} \,, \tag{3.26}$$
$$C_2 = l^2 \Lambda_{[a_1}{}^\alpha \nabla^b \mathcal{F}_{a_2 a_3} \mathcal{F}_{a_4 a_5]} (\gamma_b)_{\alpha\beta} \,. \tag{3.27}$$

Applying a spinor derivative to the remaining Js we find,

$$\Delta_2 J_3 = -i C_1 \tag{3.28}$$
$$\Delta_2 J_4 = i C_2 \,. \tag{3.29}$$

Hence the only closed combinations of Js are exact and we find the cohomology is trivial at this order in l.

We therefore proceed to higher order in l. The next possibility is l^4 and we find nineteen possible anti-self-dual 5-forms cubic in the fields. These are listed in the appendix. There are fourteen spinors $(\delta A)_\alpha$ cubic in the fields at order l^4, however, there are at most twelve independent field redefinitions because there are two combinations which can be written in the form,

$$(\delta A)_\alpha = \nabla_\alpha \phi \,, \tag{3.30}$$

for some scalar ϕ. Such spinors are exact in the sequence (3.6) and hence they give no field redefinition. By direct calculation one can verify that indeed the remaining twelve are independent. There are six possibilities in the representation (00003). By counting one can see that there is at least one J which is closed since there are 19 Js in total, twelve of which are field redefinitions, leaving 7 possibilities. There are 6 constraints from the closure condition so there is at least one solution. Again a direct calculation reveals that there is indeed only one solution, which can be written in the form,

$$J_{a_1a_2a_3a_4a_5} = l^4 \nabla_b \Lambda_c{}^\alpha \Lambda^{b\beta} \nabla^c \mathcal{F}_{[a_1a_2}(\gamma_{a_3a_4a_5]})_{\alpha\beta} - \text{dual}\,. \tag{3.31}$$

This is our main result. The above term represents the first supersymmetric and kappa-symmetric deformation of the Born-Infeld theory of the D9-brane at lowest order in fields and lowest order in the dimensionful parameter l. The details of the derivation are given in the appendix, where all the relevant quantities in the representations (00000), (00001), (00002) and (00003) are written explicitly.

4. Derivative corrections to the equations of motion

The derivative corrections to the equations of motion can be simply computed by including a non-zero $\mathcal{F}_{\alpha\beta}$ in the Bianchi identity. We work up to order three in fields since $\mathcal{F}_{\alpha\beta}$ is computed up to this order. The corrections to the various quantities given in section 2 are denoted below by a superscript (3,1), referring to order 3 in fields and order 1 in the deformation J.

The Bianchi identity at dimension $-1/2$, gives the gamma traceless part of $\mathcal{F}_{\alpha b}$ in terms of $\mathcal{F}_{\alpha\beta}$. We find

$$\hat{\mathcal{F}}_{\alpha b} = \frac{i}{10}(\gamma_b)^{\beta\gamma}\nabla_\gamma \mathcal{F}_{\alpha\beta}\,. \tag{4.1}$$

At dimension zero we find the leading corrections to the different irreps in $h_\alpha{}^\beta$.

$$h^{(3,1)} = 0\,, \tag{4.2}$$

$$h^{(3,1)}_{ab} = \frac{1}{8}(\gamma^{cde})^{\alpha\beta}\nabla_\alpha\nabla_\beta J_{abcde}\,, \tag{4.3}$$

$$h^{(3,1)}_{abcd} = -\frac{1}{24}(\gamma_{[a}{}^{ef})^{\alpha\beta}\nabla_\alpha\nabla_\beta J_{bcd]ef} - \frac{7i}{6}\nabla^e J_{abcde}\,. \tag{4.4}$$

At dimension 1/2 we have the correction to the supervariation of $\mathcal{F}$,

$$(\nabla_\alpha \mathcal{F}_{bc})^{(3,1)} = 2i(\gamma_{bc})^\gamma{}_\alpha \psi^{(3,1)}_\gamma + \frac{2i}{10}(\gamma_{[c})^{\gamma\delta}\nabla_{b]}\nabla_\gamma \mathcal{F}_{\alpha\delta}\,, \tag{4.5}$$

and the correction to the fermionic equation of motion,

$$\psi_{\alpha}^{(3,1)} = \frac{6}{10.700}\nabla_b\nabla_\epsilon\mathcal{F}_{\beta\delta}(\gamma_c)^{\epsilon\delta}(\gamma^{bc})^{\beta}{}_{\alpha} - \frac{2i}{700}(\gamma^h)^{\epsilon\beta}\nabla_\epsilon h^{(3,1)}{}_{\beta}{}^{\gamma}(\gamma_h)_{\gamma\alpha}\,. \qquad (4.6)$$

At dimension one we have no correction to the component Bianchi identity for $\mathcal{F}_{ab}$,

$$(\nabla_{[a}\mathcal{F}_{bc]})^{(3,1)} = 0\,, \qquad (4.7)$$

and the correction to the bosonic equation of motion,

$$(\nabla^a\mathcal{F}_{ab})^{(3,1)} = -\frac{10}{8}(\gamma_a)^{\alpha\epsilon}\nabla_\alpha\psi_\epsilon^{(3,1)} + \frac{1}{8}\nabla_b h^{(3,1)}{}_{\alpha}{}^{\gamma}(\gamma^b\gamma_a)_{\gamma}{}^{\alpha}\,. \qquad (4.8)$$

The correction to $\nabla_\alpha\hat{\Lambda}$ is then given by

$$(\nabla_\alpha\hat{\Lambda}_b{}^{\gamma})^{(3,1)} = -(\gamma_b)^{\gamma\delta}\nabla_\alpha\psi_\delta^{(3,1)} + \frac{1}{4}(\nabla_b\mathcal{F}_{cd})^{(3,1)}(\gamma^{cd})_{\alpha}{}^{\gamma} + \nabla_b h^{(3,1)}{}_{\alpha}{}^{\gamma}\,. \qquad (4.9)$$

This verifies that the corrections to the theory are indeed specified just by fixing the deformation of the $\mathcal{F}$-constraint which is given by J. We have calculated the first possible deformation of the constraint at order l^4 and cubic in fields and hence the first derivative deformation of the Born-Infeld theory allowed by supersymmetry.

One could also explicitly calculate the corrections to the kappa-variations of the fields. These are of the same form as in the undeformed case. For the variations of the coordinates, one simply has to bear in mind that there are corrections to the embedding matrix induced by J (4.2), (4.3), (4.4). For the variation of the gauge field one must account for the non-zero components $\mathcal{F}_{\alpha\beta}$ and $\mathcal{F}_{\alpha b}$. However, it should be emphasised that a formulation where the symmetry is manifest is preferable to one where explicit variations are required.

5. Conclusions

Using the superembedding formalism, we have shown that derivative corrections to the D9-brane effective action can be systematically computed in a manifestly kappa-symmetric manner. All supersymmetric and kappa-symmetric deformations of Born-Infeld theory can be identified with elements of a spinorial cohomology group. We have calculated explicitly the first such possible correction at leading order in the dimensionful parameter l and leading order in number of fields. In general, one expects corrections to the term we have found both at higher order in number of fields and at higher order in l in order to consistently solve the Bianchi identity. Any such higher order completion will necessarily not be unique due to the presence of higher order elements of the cohomology group. Given that some all order completion should exist, the term we have presented defines a kappa-symmetric theory which, upon gauge fixing, must reproduce the leading derivative correction to the four-point amplitude of the open superstring. It would be interesting to find an explicit form for such an invariant to all orders, although this would not be the complete effective action for the open superstring because it would not contain (for example) higher derivative four-point functions. It would also be of interest to find the correct J which would reproduce the full four-point interactions to all orders in l. One would require some input from string theory to fix the coefficients of the independent terms in such a calculation, as discussed in [8].

The constructions of [31] and the present work could be extended to branes with both gauge and transverse bosonic degrees of freedom. However, in the case of low (but not zero) codimension, there are two constraints to deform, the embedding constraint and the $\mathcal{F}$-constraint. We expect that in such cases the solution to the deformation problem is unique at order l^4.

One can make use of our results in two ways: one could assume string theory dualities, and thus construct the effective theories for other D-branes by T-duality. Alternatively, one can perform the direct calculation, using the same method to check if there is in fact only one solution, thus guaranteeing that T-duality is respected to order l^4 by supersymmetry.

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A. Details of the cohomology at l^4

We use the highest weight notation for irreducible representations. The relevant group is Spin(1, 9) which is D5 in the Cartan classification. The relevant irreducible representations are given by:

(10000) vector
(01000) two-form
(11000) traceless vector two-form
(00001) downstairs spinor
(00010) upstairs spinor
(10010) gamma-traceless (upstairs) vector spinor
(00002) anti-self dual 5-form
(00003) anti-self-dual 5-form (downstairs) spinor.

Scalars at l^4. There are two ways of constructing the representation (00000) which are cubic in the fields. They are,

$$\nabla\Lambda\Lambda\mathcal{F}: \quad \phi_1 = l^4 \nabla_b \Lambda_c{}^\alpha \Lambda^{b\beta} \mathcal{F}^{cd} (\gamma_d)_{\alpha\beta}\,, \tag{A.1}$$

$$\Lambda\Lambda\nabla\mathcal{F}: \quad \phi_2 = l^4 \Lambda_a{}^\alpha \Lambda_b{}^\beta \nabla_c \mathcal{F}^{ab} (\gamma^c)_{\alpha\beta}\,. \tag{A.2}$$

Field redefinitions at l^4. There are fourteen ways of constructing the representation (00001) which are cubic in the fields:

$$\Lambda\nabla^2\mathcal{F}\mathcal{F}: \quad (\delta A_1)_\alpha = l^4 \Lambda_a{}^\beta \nabla^a \nabla^b \mathcal{F}^{cd} \mathcal{F}_{bc} (\gamma_d)_{\alpha\beta}\,, \tag{A.3}$$

$$(\delta A_2)_\alpha = l^4 \Lambda_a{}^\beta \nabla^a \nabla^b \mathcal{F}^{cd} \mathcal{F}_b{}^e (\gamma_{cde})_{\alpha\beta}\,, \tag{A.4}$$

$$\nabla\Lambda\nabla\mathcal{F}\mathcal{F}:\ (\delta A_3)_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \nabla^a \mathcal{F}^{bc} \mathcal{F}_{cd} (\gamma^d)_{\alpha\beta}\,, \tag{A.5}$$

$$(\delta A_4)_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \nabla^a \mathcal{F}_{cd} \mathcal{F}^{bc} (\gamma^d)_{\alpha\beta}\,, \tag{A.6}$$

$$(\delta A_5)_\alpha = l^4 \nabla_a \Lambda^{b\beta} \nabla_c \mathcal{F}^{ad} \mathcal{F}_{bd} (\gamma^c)_{\alpha\beta}\,, \tag{A.7}$$

$$(\delta A_6)_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \nabla^a \mathcal{F}^{bc} \mathcal{F}^{de} (\gamma_{cde})_{\alpha\beta}\,, \tag{A.8}$$

$$(\delta A_7)_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \nabla^a \mathcal{F}^{cd} \mathcal{F}^{be} (\gamma_{cde})_{\alpha\beta}\,, \tag{A.9}$$

$$\Lambda\nabla\mathcal{F}\nabla\mathcal{F}:\ (\delta A_8)_\alpha = l^4 \Lambda_a{}^\beta \nabla^a \mathcal{F}_{bc} \nabla^b \mathcal{F}^{cd} (\gamma_d)_{\alpha\beta}\,, \tag{A.10}$$

$$(\delta A_9)_\alpha = l^4 \Lambda_a{}^\beta \nabla_b \mathcal{F}^{ac} \nabla_c \mathcal{F}^{bd} (\gamma_d)_{\alpha\beta}\,, \tag{A.11}$$

$$(\delta A_{10})_\alpha = l^4 \Lambda_a{}^\beta \nabla_b \mathcal{F}^{ca} \nabla_c \mathcal{F}_{de} (\gamma^{bde})_{\alpha\beta}\,, \tag{A.12}$$

$$(\delta A_{11})_\alpha = l^4 \Lambda_a{}^\beta \nabla^a \mathcal{F}_{bc} \nabla^c \mathcal{F}_{de} (\gamma^{bde})_{\alpha\beta}\,, \tag{A.13}$$

$$\nabla\Lambda\Lambda\Lambda:\ (\delta A_{12})_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \Lambda^{a\gamma} \Lambda^{c\delta} (\gamma_c)_{\alpha\beta} (\gamma^b)_{\gamma\delta}\,, \tag{A.14}$$

$$(\delta A_{13})_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \Lambda^{a\gamma} \Lambda^{b\delta} (\gamma_{cde})_{\alpha\beta} (\gamma^{cde})_{\gamma\delta}\,, \tag{A.15}$$

$$(\delta A_{14})_\alpha = l^4 \nabla_a \Lambda_b{}^\beta \Lambda^{a\gamma} \Lambda^{c\delta} (\gamma_{cde})_{\alpha\beta} (\gamma^{bde})_{\gamma\delta}\,. \tag{A.16}$$

Two linear combinations of these are exact in the sequence (3.6). One finds,

$$\Delta_0 \phi_1 = -\frac{1}{2}(\delta A_1) - \frac{1}{4}(\delta A_2) + \frac{1}{2}(\delta A_4) + \frac{1}{4}(\delta A_7) - \frac{i}{4}(\delta A_{12}) + \frac{i}{8}(\delta A_{14})\,, \tag{A.17}$$

$$\Delta_0 \phi_2 = (\delta A_9) - \frac{1}{2}(\delta A_{10}) + 2i(\delta A_{12})\,. \tag{A.18}$$

We can use these relations to eliminate (δA_{10}) and (δA_{14}) when computing which combinations of Js are exact in the sequence (3.6).

Deformations at l^4. There are nineteen ways of constructing the representation (00002) which are cubic in the fields. They are given below as tensors $J_{a_1 a_2 a_3 a_4 a_5}$. It is understood that one antisymmetrises over the free indices $a_1 \cdots a_5$ and explicitly subtracts the dual.

$$\nabla^2\Lambda\Lambda\mathcal{F}:\ J_1 = l^4 \nabla_b \nabla_c \Lambda_{a_1}{}^\alpha \Lambda^{b\beta} \mathcal{F}^c{}_{a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.19}$$

$$\Lambda\Lambda\nabla^2\mathcal{F}:\ J_2 = l^4 \Lambda_b{}^\alpha \Lambda_c{}^\beta \nabla^b \nabla^c \mathcal{F}_{a_1 a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.20}$$

$$J_3 = l^4 \Lambda_b{}^\alpha \Lambda_{a_1}{}^\beta \nabla^b \nabla^c \mathcal{F}_{a_2 a_3} (\gamma_{a_4 a_5 c})_{\alpha\beta}\,, \tag{A.21}$$

$$J_4 = l^4 \Lambda_b{}^\alpha \Lambda_c{}^\beta \nabla^d \nabla_{a_1} \mathcal{F}^{bc} (\gamma_{a_2 a_3 a_4 a_5 d})_{\alpha\beta}\,, \tag{A.22}$$

$$\nabla\Lambda\nabla\Lambda\mathcal{F}:\ J_5 = l^4 \nabla_b \Lambda_{a_1}{}^\alpha \nabla^b \Lambda_{a_2}{}^\beta \mathcal{F}_{a_3 a_4} (\gamma_{a_5})_{\alpha\beta}\,, \tag{A.23}$$

$$J_6 = l^4 \nabla_b \Lambda_c{}^\alpha \nabla^b \Lambda^{c\beta} \mathcal{F}_{a_1 a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.24}$$

$$J_7 = l^4 \nabla_b \Lambda_c{}^\alpha \nabla^b \Lambda_{a_1}{}^\beta \mathcal{F}^c{}_{a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.25}$$

$$J_8 = l^4 \nabla_b \Lambda_{a_1}{}^\alpha \nabla_c \Lambda_{a_2}{}^\beta \mathcal{F}^{bc} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.26}$$

$$\nabla\Lambda\Lambda\nabla\mathcal{F}:\ J_9 = l^4 \nabla_b \Lambda_{a_1}{}^\alpha \Lambda_{a_2}{}^\beta \nabla^b \mathcal{F}_{a_3 a_4} (\gamma_{a_5})_{\alpha\beta}\,, \tag{A.27}$$

$$J_{10} = l^4 \nabla_b \Lambda_c{}^\alpha \Lambda^{b\beta} \nabla^c \mathcal{F}_{a_1 a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.28}$$

$$J_{11} = l^4 \nabla_b \Lambda_c{}^\alpha \Lambda_{a_1}{}^\beta \nabla^b \mathcal{F}^c{}_{a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.29}$$

$$J_{12} = l^4 \nabla_b \Lambda_{a_1}{}^\alpha \Lambda_{a_2}{}^\beta \nabla^b \mathcal{F}^c{}_{a_3} (\gamma_{a_4 a_5 c})_{\alpha\beta}\,, \tag{A.30}$$

$$J_{13} = l^4 \nabla_b \Lambda_{a_1}{}^\alpha \Lambda_c{}^\beta \nabla^b \mathcal{F}^c{}_{a_2} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}\,, \tag{A.31}$$

$$J_{14} = l^4 \nabla_b \Lambda_{a_1}{}^{\alpha} \Lambda_c{}^{\beta} \nabla_{a_2} \mathcal{F}^{bc} (\gamma_{a_3 a_4 a_5})_{\alpha\beta}, \tag{A.32}$$
$$J_{15} = l^4 \nabla_b \Lambda_{a_1}{}^{\alpha} \Lambda^{b\beta} \nabla^c \mathcal{F}_{a_2 a_3} (\gamma_{a_4 a_5 c})_{\alpha\beta}\,, \tag{A.33}$$
$$J_{16} = l^4 \nabla_b \Lambda_c{}^{\alpha} \Lambda^{b\beta} \nabla^d \mathcal{F}^c{}_{a_1} (\gamma_{a_2 a_3 a_4 a_5 d})_{\alpha\beta}\,, \tag{A.34}$$
$$\nabla^2 \mathcal{F} \nabla \mathcal{F} \mathcal{F}: \quad J_{17} = l^4 \nabla_b \nabla_c \mathcal{F}_{a_1 a_2} \nabla^b \mathcal{F}^c{}_{a_3} \mathcal{F}_{a_4 a_5}\,, \tag{A.35}$$
$$J_{18} = l^4 \nabla_b \nabla_c \mathcal{F}_{a_1 a_2} \nabla^b \mathcal{F}_{a_3 a_4} \mathcal{F}^c{}_{a_5}\,, \tag{A.36}$$
$$\nabla \mathcal{F} \nabla \mathcal{F} \nabla \mathcal{F}: \quad J_{19} = l^4 \nabla_b \mathcal{F}_{c a_1} \nabla^b \mathcal{F}_{a_2 a_3} \nabla^c \mathcal{F}_{a_4 a_5}\,. \tag{A.37}$$

From the twelve independent spinors given previously we derive the following twelve exact combinations of the above Js which correspond to field redefinitions and not genuine deformations,

$$E_1 = \Delta_1(\delta A_1) = 16 J_1 + 4 J_2 + J_4\,, \tag{A.38}$$
$$E_2 = \Delta_1(\delta A_2) = 96 J_{18} + i[16 J_1 + 4 J_2 - 48 J_3 + J_4]\,, \tag{A.39}$$
$$E_3 = \Delta_1(\delta A_3) = J_6 - 2 J_7 + 2 J_{11}\,, \tag{A.40}$$
$$E_4 = \Delta_1(\delta A_4) = 2 J_7 + J_{10} - 2 J_{13}\,, \tag{A.41}$$
$$E_5 = \Delta_1(\delta A_5) = 8 J_7 + 8 J_8 - 2 J_{10} + 8 J_{14} + J_{16}\,, \tag{A.42}$$
$$E_6 = \Delta_1(\delta A_6) = -96 J_{17} + i[\cdots]\,, \tag{A.43}$$
$$E_7 = \Delta_1(\delta A_7) = 6 J_{18} + i[J_7 - 6 J_9 - J_{11} + 3 J_{12}]\,, \tag{A.44}$$
$$E_8 = \Delta_1(\delta A_8) = 2 J_{10} + 8 J_{13} + 8 J_{14} + J_{16}\,, \tag{A.45}$$
$$E_9 = \Delta_1(\delta A_9) = 2 J_{10} - 8 J_{11} - 8 J_{14} - J_{16}\,, \tag{A.46}$$
$$E_{10} = \Delta_1(\delta A_{11}) = J_{19} + i[\cdots]\,, \tag{A.47}$$
$$E_{11} = \Delta_1(\delta A_{12}) = 6 J_9 - J_{11} + 3 J_{12} + J_{13} - 3 J_{15}\,, \tag{A.48}$$
$$E_{12} = \Delta_1(\delta A_{13}) = J_2 - J_{10}\,. \tag{A.49}$$

The notation $[\ldots]$ is used above to denote a linear combination of terms involving Λ (the terms J_1 to J_{16}). The details of the linear combination are not needed for the ensuing analysis. The above relations mean that it is consistent to remove all but J_3, J_4, J_5, J_9, J_{10}, J_{14}, J_{16} from the set of Js to check which satisfy closure non-trivially.

Closure constraints at l^4. There are six ways of constructing the representation (00003) which are cubic in the fields. They are given below as tensors, $C^{a_1 a_2 a_3 a_4 a_5}{}_{\delta}$. It is understood that one should antisymmetrise the free indices $a_1 \cdots a_5$, explicitly subtract the dual and take the γ-traceless part.

$$\nabla\Lambda\nabla\Lambda\Lambda: \quad C_1 = l^4 \nabla_b \Lambda^{a_1 \alpha} \nabla_c \Lambda^{a_2 \beta} \Lambda^{c\gamma} (\gamma^{a_3 a_4 a_5})_{\alpha\beta} (\gamma^b)_{\gamma\delta} \tag{A.50}$$
$$C_2 = l^4 \nabla_b \Lambda_c{}^{\alpha} \nabla^c \Lambda^{a_1 \beta} \Lambda^{a_2 \gamma} (\gamma^{a_3 a_4 a_5})_{\alpha\beta} (\gamma^b)_{\gamma\delta} \tag{A.51}$$
$$\Lambda\Lambda\nabla^2\Lambda: \quad C_3 = l^4 \Lambda_b{}^{\alpha} \Lambda^{a_1 \beta} \nabla^b \nabla_c \Lambda^{a_2 \gamma} (\gamma^{c d a_3 a_4 a_5})_{\alpha\beta} (\gamma_d)_{\gamma\delta} \tag{A.52}$$
$$\nabla\Lambda\nabla^2\mathcal{F}\mathcal{F}: \quad C_4 = l^4 \nabla_b \Lambda^{a_1 \alpha} \nabla^b \nabla^c \mathcal{F}^{a_2 a_3} \mathcal{F}^{a_4 a_5} (\gamma_c)_{\alpha\delta} \tag{A.53}$$
$$\nabla\Lambda\nabla\mathcal{F}\nabla\mathcal{F}: \quad C_5 = l^4 \nabla_b \Lambda^{a_1 \alpha} \nabla^b \mathcal{F}^{a_2 a_3} \nabla^c \mathcal{F}^{a_4 a_5} (\gamma_c)_{\alpha\delta} \tag{A.54}$$
$$\Lambda\nabla^2\mathcal{F}\nabla\mathcal{F}: \quad C_6 = l^4 \Lambda^{a_1 \alpha} \nabla_b \nabla_c \mathcal{F}^{a_2 a_3} \nabla^b \mathcal{F}^{a_4 a_5} (\gamma^c)_{\alpha\delta}\,. \tag{A.55}$$

Applying Δ_2 to the nineteen Js we find:

$$\begin{array}{lll}
\Delta_2 J_1 = \frac{i}{2} C_3 & \Delta_2 J_8 = -2iC_1 & \Delta_2 J_{15} = -2C_5 \\
\Delta_2 J_2 = 0 & \Delta_2 J_9 = -\frac{1}{2}(C_5 + C_6) & \Delta_2 J_{16} = -8iC_1 \\
\Delta_2 J_3 = -2C_6 & \Delta_2 J_{10} = 0 & \Delta_2 J_{17} = -iC_4 \\
\Delta_2 J_4 = -8iC_3 & \Delta_2 J_{11} = -iC_2 & \Delta_2 J_{18} = -iC_6 \\
\Delta_2 J_5 = C_4 & \Delta_2 J_{12} = C_6 - C_5 & \Delta_2 J_{19} = -iC_5 \\
\Delta_2 J_6 = 0 & \Delta_2 J_{13} = -iC_2 & \\
\Delta_2 J_7 = -iC_2 & \Delta_2 J_{14} = i(C_1 + C_2) &
\end{array} \tag{A.56}$$

Thus we can see that from the set, $\{J_3, J_4, J_5, J_9, J_{10}, J_{14}, J_{16}\}$, the only closed J is given by J_{10}. This is the fist non-trivial deformation of the $\mathcal{F}$-constraint.

In constructing the terms in the preceding sections the program LiE, [38] proved useful. We used it to check that we found the correct number of terms of each representation. We have also checked some of the gamma-matrix manipulations with the Mathematica package GAMMA [39].

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